Chapter 10 - Day 1

Previously, if given $f(x)$, we found the derivative $f'(x)$. In this chapter, when given $f(x)$, we'll find $F(x)$ Such that $F'(x) = f(x)$. $F(x)$ is called the <u>antiderivative</u> of $f(x)$.

<u>Ex</u>: Find $(x^3)'$

$$(x^3)' = 3x^2$$

Thus the antiderivative of $f(x) = 3x^2$ is $F(x) = x^3$

<u>BUT</u> $F(x) = x^3 + 6$ is also an antiderivative of $f(x)$ since $F'(x) = 3x^2 = f(x)$

* Therefore if $F(x)$ is an antiderivative of $f(x)$, then so is $F(x)+c$ where c is a constant.

The <u>indefinite integral</u> of $f(x)$, denoted $\int f(x)\,dx$ without limits of integration, is the general antiderivative of $f(x)$.

Thus $\int 3x^2\,dx = x^3 + C$

where c is a constant.

Ex: if $F(x) = 3x^4$, what is $F'(x)$?

$$F'(x) = \boxed{12x^3}$$

thus, what is $\int 12x^3\,dx$?

$$\int 12x^3\,dx = \boxed{3x^4 + C}$$

think about the power rule....
how do we "undo" it?

$$\int a x^b\,dx = \frac{a}{b+1} x^{b+1} + C$$

Some Basic Indefinite Integrals

① $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c \quad (n \neq -1)$

② $\int e^x \, dx = e^x + c$

③ $\int \frac{1}{x} \, dx = \ln|x| + c$

Rules for Indefinite Integrals

(A) $\int c f(x)\,dx = c\int f(x)\,dx$

(B) $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Ex: Evaluate $\int (t^3 + 3t^2 + 4t + 9)\, dt$

$$= \int t^3\, dt + \int 3t^2\, dt + \int 4t\, dt + \int 9\, dt$$

$$= \int t^3\, dt + 3\int t^2\, dt + 4\int t\, dt + 9\int \underset{\overset{\shortparallel}{t^0}}{1}\, dt$$

$$= \left(\frac{1}{4}t^4 + c\right) + 3\left(\frac{1}{3}t^3 + c\right) + 4\left(\frac{1}{2}t^2 + c\right) + 9\left(\frac{1}{1}t^1 + c\right)$$

$$= \boxed{\frac{1}{4}t^4 + t^3 + 2t^2 + 9t + c}$$

Ex: Evaluate $\int \frac{6}{\sqrt{t}}\, dt$

$$\int \frac{6}{\sqrt{t}}\, dt = \int 6t^{-1/2}\, dt$$

$$= 6\int t^{-1/2}\, dt$$

$$= 6\left(\frac{1}{1/2} t^{1/2}\right) + c$$

$$= 12t^{1/2} + c$$

$$= \boxed{12\sqrt{t} + c}$$

Ex: Evaluate $\int t^3 (t+2)\,dt$

$$= \int (t^4 + 2t^3)\,dt$$

$$= \int t^4\,dt + \int 2t^3\,dt$$

$$= \int t^4\,dt + 2\int t^3\,dt$$

$$= \frac{1}{5}t^5 + 2\left(\frac{1}{4}t^4\right) + C$$

$$= \boxed{\frac{1}{5}t^5 + \frac{1}{2}t^4 + C}$$

Ex: Evaluate $\int \frac{x^2+9}{x^2}\,dx$

$$\frac{x^2+9}{x^2} = \frac{x^2}{x^2} + \frac{9}{x^2}$$

$$\int \frac{x^2+9}{x^2}\,dx = \int \frac{x^2}{x^2}\,dx + \int \frac{9}{x^2}\,dx$$

$$= \int 1\,dx + 9\int x^{-2}\,dx$$

$$= x + 9\left(\frac{1}{-1}x^{-1}\right) + C$$

$$= x - 9x^{-1} + C$$

$$= \boxed{x - \frac{9}{x} + C}$$